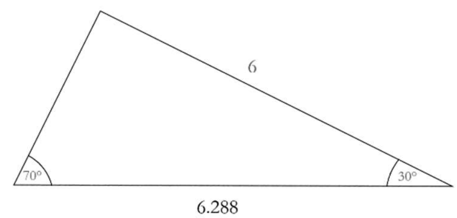
The Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

This equation can also be written $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$, and is used to find an unknown angle or an unknown side of a triangle. As the values a and A could represent any side of a triangle and angle of a triangle opposite that side respectively $\sin A$ and $\sin B$ mean the function of $\sin$ is used on angles A and B.

For example, let $a = 6$ - meaning the angle opposite this length is A and in this example $A = 70°$. Let $B = 30°$, so that we can solve to find b, which is the unmarked side on this triangle to the right.

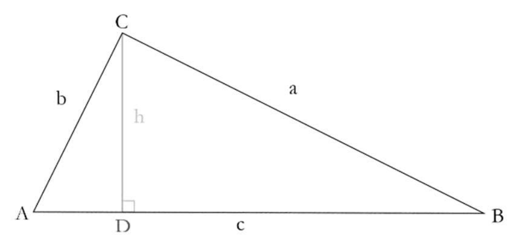


Using this format of the equation as it can be more easily re-arranged

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{6}{\sin 70°} = \frac{b}{\sin 30°}$$
$$\frac{6}{\sin 70°} \times \sin 30° = b$$
$$b = 3.1925 \ldots$$

Proof

Consider the triangle to the right. As we can see, ΔACD and ΔCBD are both right-angled triangles. This means we can apply SOH CAH TOA to both of these smaller triangles that make up the larger triangle.



Using SOH CAH TOA

$$\sin \theta = \frac{opposite}{hypotenuse}$$

For ΔACD

$$\sin A = \frac{h}{b}$$

Re-arranging this gives

$$h = b \sin A$$

For ΔCBD

$$\sin B = \frac{h}{a}$$

Re-arranging this gives

$$h = a \sin B$$

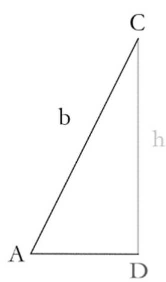
Equating the two expressions that equal h

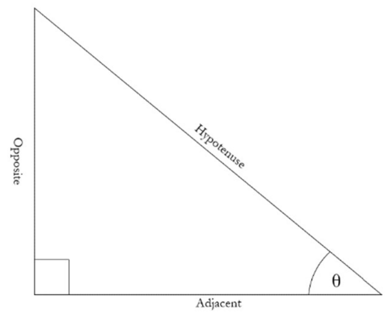
$$b \sin A = a \sin B$$

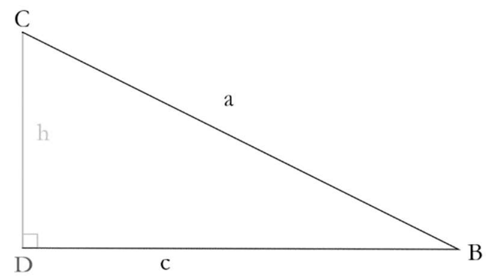
Re-arranging this by cross multiplication

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

For reference, here is the general form for a right-angled triangle between the triangles ACD and CBD used to find $\sin \theta$ in step 1 and $\sin A$ and $\sin B$ in steps 2 and 4.







Note

Consider the equation $\sin x = 0.5$

Of course, a solution this is $x = 30°$, but another solution is $x = 150°$. When using the Sine Rule to find an angle, there will always be two possible angles that solve the equation (unless if the solution is $90°$).

For example, for the triangle on the right we can let $a = 3, b = 4$ and therefore $A = 45°$.

Using the Sine Rule*

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

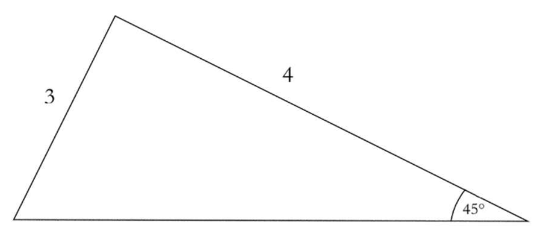
$$\frac{\sin 45°}{3} = \frac{\sin B}{4}$$

$$\frac{\sin 45°}{3} \times 4 = \sin B$$

$$\sin B = 0.9428 \ldots$$

$$B = \sin^{-1} 0.9428 \ldots$$

$$B = 70.52 \ldots or\ 109.47 \ldots$$



Although it may seem obvious that the angle B is acute from the diagram, you cannot prove it is $70.52 \ldots °$ as opposed to $109.47 \ldots °$, but usually in an exam there will be more information which may allow you to use the Cosine Rule, for which the problem does not occur when calculating angles.

*N.B. Here we use the other form of the sine rule, compared to the previous example. It is usually easiest to have the unknown value (in this case the angle B) as the numerator.

See also

- SOH CAH TOA
- Sine Area Rule
- Cosine Rule

References

Attwood, G. et al. (2017). *Edexcel AS and A level Mathematics - Pure - Year 1*. London: Pearson Education. pp.179-180